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BRITTLE FRACTURE OF A CORE WHEN DRILLING IN A COMPRESSED MEDIUM

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When drilling with core sampling in a medium compressed by mountain pressure, fracture of the core into separate disks is usually observed [1]. The thickness of the disks formed is related to the value of the mountain pressure, and an increase in the pressure causes a reduction in the thickness of the core disks which have split off. This experimentally established relationship is the basis of one of the methods used to determine impact-dangerous parts in mines [2].

In this paper this phenomenon is investigated theoretically using the model of an ideally elastic medium, fractured in a brittle manner. The following assumptions are made: a) The thickness of the walls of the drilling instrument is assumed to be zero, as also the distance between the edges of the cylindricals cavity drilled out by the drill in the rock; b) the action of the drill on the core during drilling is described by a distributed tangential stress which twists the core. The normal stresses on the sides of the cut are assumed to be zero; c) uniform compression stresses act at infinity, perpendicular to the axis of the cylindrical crack.

With these assumptions the problem of the fracture of a core in this model reduces to an analysis of the stressed state in the region of the edge of the cylindrical cut produced, and, more accurately, to a determination of the intensity coefficients of the stress field K_I, K_{II}, and K_{III} [3].

The simplest problem of the equilibrium in an infinite isotropic elastic space of a cylindrical cut of radius a and length $2l$ whose axis is along the z axis, as shown in Fig. 1, is considered. Two cases of loading are considered: compression transverse to the z axis by a pressure equal to p_0 at infinity, and twisting along the axis by a stress applied to the surface of the core.

1. The Axisymmetric Case. We will assume that the displacement vector is independent of the angle φ and has the form $\mathbf{u} = u\cdot\mathbf{r} + w\cdot\mathbf{z}$. We will introduce dimensionless quantities by the equations (henceforth, for simplicity, the primes will be omitted)

$$\langle u, w, z, r, a\rangle' = \frac{\langle u, w, z, r, a\rangle}{l},$$

$$\sigma'_{ij} = \sigma_{ij}/\mu, \quad \langle p_0, \tau_0\rangle' = \langle p_0, \tau_0\rangle/\mu.$$

Then the equations of equilibrium and the components of the stress tensor can be written in the form

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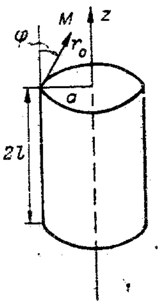

Fig. 1

$$2(1-\nu)\left[\frac{\partial^2 u}{\partial r^2}+\frac{1}{r}\frac{\partial u}{\partial r}-\frac{u}{r^2}\right]+(1-2\nu)\frac{\partial^2 u}{\partial z^2}+\frac{\partial^2 w}{\partial z\,\partial r}=0, \tag{1.1}$$

$$(1-2\nu)\left[\frac{\partial^2 w}{\partial r^2}+\frac{1}{r}\frac{\partial w}{\partial r}\right]+2(1-\nu)\frac{\partial^2 w}{\partial z^2}+\frac{\partial}{\partial z}\left[\frac{\partial u}{\partial r}+\frac{u}{r}\right]=0,$$

$$\sigma_{rr}=(1-2\nu)^{-1}[(1-\nu)\partial u/\partial r+\nu(u/r+\partial w/\partial z)],$$

$$\sigma_{zz}=(1-2\nu)^{-1}[(1-\nu)\partial w/\partial z+\nu(u/r+\partial u/\partial r)],\quad \sigma_{rz}=2^{-1}[\partial u/\partial z+\partial w/\partial r].$$

We will distinguish two regions in the space considered. Quantities relating to the region $r \le a$ will be given the subscript 1, and those relating to the region $r \ge a$ will be given the subscript 2. Assuming the deformed state of the body to be symmetrical with respect to the $z = 0$ plane, we will write the general solution of the equilibrium equations (1.1) as follows:

$$u_1(r,z)=\frac{2}{\pi}\int_0^\infty \{srA(s)I_0(sr)+[sB(s)-4(1-\nu)A(s)]I_1(sr)\}\cos(sz)\,ds,$$

$$w_1(r,z)=-\frac{2}{\pi}\int_0^\infty s[B(s)I_0(sr)+rA(s)I_1(sr)]\sin(sz)\,ds,$$

$$u_2(r,z)=\frac{2}{\pi}\int_0^\infty \{srC(s)K_0(sr)+[sD(s)+4(1-\nu)C(s)]K_1(sr)\}\cos(sz)\,ds,$$

$$w_2(r,z)=\frac{2}{\pi}\int_0^\infty s[D(s)K_0(sr)+rC(s)K_1(sr)]\sin(sz)\,ds.$$

The four arbitrary functions $A(s)$, $B(s)$, $C(s)$, $D(s)$ can be found from the boundary conditions

$$\sigma_{rr}^{(1)}=p_0p(z),\quad \sigma_{rz}^{(1)}=0\quad \text{for}\quad r=a,\ |z|\leqslant 1 \tag{1.2}$$

and the additional conditions (the continuity of the displacement field and the continuity of the components of the stress tensor on the surface $r = a$)

$$u_1=u_2,\quad w_1=w_2\quad \text{for}\quad |z|\geqslant 1,$$
$$\sigma_{rr}^{(1)}=\sigma_{rr}^{(2)},\ \sigma_{rz}^{(1)}=\sigma_{rz}^{(2)}\quad \text{for}\quad |z|<\infty. \tag{1.3}$$

From the second pair of conditions (1.3) we obtain expressions for $A(s)$ and $C(s)$ in terms of the two unknown functions $B(s)$ and $D(s)$. After this the remaining conditions (1.3) and the boundary conditions (1.2) can be represented in the form

$$\frac{1}{2(1-\nu)}[u_1-u_2]=\frac{2}{\pi}\int_0^\infty [B_1(s)f_9+D_1(s)f_{10}]\cos(sz)\,ds=0,\quad z\geqslant 1,$$

$$\frac{1}{2(1-\nu)}\frac{\partial}{\partial z}[w_1-w_2]=\frac{2}{\pi}\int_0^\infty s[B_1(s)f_{11}+D_1(s)f_{12}]\cos(sz)\,ds=0,$$

$$\sigma_{rr}^{(1)}=\frac{2}{\pi}\int_0^\infty [B_1(s)F_1-D_1(s)F_2]\cos(sz)\,ds=p_0p(z), \tag{1.4}$$

$$\sigma_{rz}^{(1)}=\frac{2}{\pi}\int_0^\infty [B_1(s)F_3+D_1(s)F_4]\,s\sin(sz)\,ds=0,\quad 0\leqslant z\leqslant 1,$$

where $B_1(s) = B(s)\,sF_0^{-1}$; $D_1(s) = D(s)\,sF_0^{-1}$;

$$F_0 = f_3f_5 + f_7f_1;\ F_1 = s^{-1}f_3[f_5f_2 - f_6f_1];$$

$$F_2 = s^{-1}f_1[f_4\ f_7 - f_3f_8];\ F_3 = F_1f_7f_3^{-1};\ F_4 = F_2f_5f_1^{-1};$$

$$f_1 = -(3-2\nu)\,sI_0(sa) + \left[s^2a + \frac{4(1-\nu)}{a}\right]I_1(sa);$$

$$f_2 = s^2aI_0(sa) - \frac{s}{a}I_1(sa);$$

$$f_3 = (3-2\nu)\,sK_0(sa) + \left[s^2a + \frac{4(1-\nu)}{a}\right]K_1(sa);\ f_4 = s^2aK_0(sa) + \frac{s}{a}K_1(sa);$$

$$f_5 = saI_0(sa) - 2(1-\nu)I_1(sa);\ f_6 = sI_1(sa);$$

$$f_7 = saK_0(sa) + 2(1-\nu)K_1(sa);\ f_8 = sK_1(sa);$$

$$f_9 = sI_0(sa) + \frac{2(1-\nu)}{a}I_1(sa);\ f_{10} = sK_0(sa) - \frac{2(1-\nu)}{a}K_1(sa);$$

$$f_{11} = \frac{1-2\nu}{a}I_0(sa) + f_6;\ f_{12} = \frac{1-2\nu}{a}K_0(sa) - f_8.$$

We will reduce the system of four integral equations obtained for the two unknown functions $B_1(s)$ and $D_1(s)$ to a system of two integral Fredholm equations of the second kind for the function $\varphi(t)$ and $\psi(t)$, continuous in the interval [0, 1] by the method described in [4].

The functions $\varphi(t)$ and $\psi(t)$ are specified by the equations

$$u_0(a, z) = \int_z^1 \frac{\tau\varphi(\tau)\,d\tau}{\sqrt{\tau^2 - z^2}},\quad w_0(a, z) = \frac{\delta}{\sqrt{1-z^2}} + \int_z^1 \frac{\psi(\tau)\,d\tau}{\sqrt{\tau^2 - z^2}},$$

where $u_0(a, z)$ and $w_0(a, z)$ are identical with $u_1(a, z) - u_2(a, z)$ and $\partial w_1/\partial z - \partial w_2/\partial z$ for $r = a$, apart from the coefficients. The parameter δ is found from the condition that the displacements $w_1(a, z)$ and $w_2(a, z)$ should be identical as $z \to 1$, and is given by the expression

$$\delta = -\int_0^1 \psi(t)\,dt.$$

From the first two equations of system (1.4) we obtain equations defining $B_1(s)$ and $D_1(s)$ in terms of the functions $\varphi(t)$ and $\psi(t)$

$$B_1(s) = F^{-1}[-s^{-1}f_{10}\Psi_0 + f_{12}\Phi_0],\ D_1(s) = F^{-1}[s^{-1}f_9\Psi_0 - f_{11}\Phi_0],$$

where

$$F = -s/a + \frac{2(1-\nu)(1-2\nu)}{sa^3};\ \Phi_0 = \frac{\pi}{2}\int_0^1 \tau\varphi(\tau)\,J_0(s\tau)\,d\tau;$$

$$\Psi_0 = \frac{\pi}{2}\left[\delta J_0(s) + \int_0^1 \psi(\tau)\,J_0(s\tau)\,d\tau\right].$$

The remaining equations of system (1.4) can be reduced, as in [4], to a system of integral Fredholm equations of the second kind (for simplicity we take $p(z) = 1 = \text{const}$)

$$\begin{aligned} &\varphi_1(t) + 2\int_0^1 \varphi_1(\tau)\,K_1(\tau, t)\,d\tau - 2\int_0^1 \psi_1(\tau)\,K_2(\tau, t)\,d\tau = 2\sqrt{t},\\ &\psi_1(t) + 2\int_0^1 \psi_1(\tau)\,K_3(\tau, t)\,d\tau + 2\int_0^1 \varphi_1(\tau)\,K_4(\tau, t)\,d\tau = 4\frac{\alpha}{a}\sqrt{t}, \end{aligned} \tag{1.5}$$

where

$$\varphi_1(t) = t^{1/2}\varphi(t)\,p_0^{-1};\ \psi_1(t) = t^{-1/2}\psi(t)\,p_0^{-1};\ \delta = -\int_0^1 \sqrt{t}\,\psi(t)\,dt;$$

$$K_1(\tau, t) = \sqrt{\tau t}\int_0^\infty sg_1(s)\,J_0(st)\,J_0(s\tau)\,ds;$$

$$K_2(\tau, t) = \sqrt{\tau t}\int_0^\infty sg_2(s)\,J_0(st)\,[J_0(s\tau) - J_0(s)]\,ds;$$

$$K_3(\tau, t) = \sqrt{\tau t}\int_0^\infty s\left[g_4(s) - \frac{2\alpha}{a} g_2(s)\right] J_0(st)\,[J_0(s\tau) - J_0(s)]\,ds;$$

$$K_4(\tau, t) = \sqrt{\tau t}\int_0^\infty s\left[g_3(s) - \frac{2\alpha}{a} g_1(s)\right] J_0(st)\,J_0(s\tau)\,ds;$$

$$g_1(s) = [z^2 + (3 - 2\nu)]\,\Phi_1 + \frac{2}{z}\Phi_2 - 1/2; \quad g_2(s) = -s^{-1}[z\Phi_1 + \Phi_2];$$

$$g_3(s) = s\,[z\Phi_1 + \Phi_2] + \frac{\alpha}{a};$$

$$g_4(s) = z^2\Phi_1 - 1/2; \quad \Phi_1 = I_0(z)K_1(z) - I_1(z)K_0(z); \quad \Phi_2 = z^2 I_0(z)K_0(z)$$
$$- [z^2 + 2(1-\nu)]I_1(z)K_1(z); \; z = sa; \; \alpha = 1/4 - \nu.$$

By finding a solution of system (1.5) we can calculate the stress fields and the displacements at any point of our region.

2. Twisting of a Cylindrical Crack. In this case the unit component of the displacement vector $u_\varphi = u(r, z)$ differs from zero, while the equilibrium equation and the nonzero components of the stress tensor have the form (in dimensionless form)

$$\frac{\partial^2 u}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} = 0, \qquad \sigma_{r\varphi} = \partial u/\partial r - u/r, \quad \sigma_{z\varphi} = \partial u/\partial z. \tag{2.1}$$

We will assume that $u(r, z)$ is an even function of z. As previously, the subscript 1 will denote quantities relating to the inner cylinder while the subscript 2 will relate to the external cylinder $r \geq a$. The solution of the equilibrium equation (2.1) can then be written in the form

$$u^{(1)}(r, z) = \frac{2}{\pi}\int_0^\infty A(s)\,I_1(sr)\cos(sz)\,ds, \quad u^{(2)}(r, z) = \frac{2}{\pi}\int_0^\infty B(s)\,K_1(sr)\cos(sz)\,ds.$$

The arbitrary functions $A(s)$ and $B(s)$ are found from the boundary conditions and the continuity conditions for $r = a$

$$\begin{gathered} \sigma_{r\varphi}^{(1)} = \tau_0\tau_1(z), \quad \sigma_{r\varphi}^{(2)} = \tau_0\tau_2(z) \quad \text{for } |z| \leqslant 1, \\ \sigma_{r\varphi}^{(1)} = \sigma_{r\varphi}^{(2)}, \quad u^{(1)} = u^{(2)} \quad \text{for } |z| \geqslant 1. \end{gathered} \tag{2.2}$$

The first three conditions define the expression $\sigma_{r\varphi}^{(1)}(a, z) - \sigma_{r\varphi}^{(2)}(a, z)$ for $r = a$, $|z| < \infty$, whence we have

$$A(s)I_2(sa) + B(s)K_2(sa) = T_0(s),$$

where

$$T_0(s) = s^{-1}\tau_0\int_0^1 [\tau_1(z) - \tau_2(z)]\cos(sz)\,dz.$$

Introducing the new function $F(s)$ defined by the equation

$$A(s)I_1(as) - B(s)K_1(as) = F(s),$$

the first and last conditions from (2.2) can be written in the form

$$\begin{gathered} \frac{2}{\pi}\int_0^\infty s^2 a I_2(sa)\,[K_2(sa)\,F(s) + T_0(s)\,K_1(sa)]\cos(sz)\,ds = \tau_0\,\tau_1(z), \quad 0 \leqslant z \leqslant 1, \\ w(a, z) = u^{(1)}(a, z) - u^{(2)}(a, z) = \frac{2}{\pi}\int_0^\infty F(s)\cos(sz)\,ds = 0, \quad z \geqslant 1. \end{gathered} \tag{2.3}$$

From the last equation we obtain

$$F(s) = \int_0^1 w(a, z)\cos(sz)\,dz. \tag{2.4}$$

Since the displacements $u^{(i)}(a, z)$ $(i = 1, 2)$ as $z \to 1$, i.e., at the head of the crack, behaves as $(1 - z)^{1/2}$, we will introduce the function $\varphi(t)$ by the equation

$$w(a, z) = \int_z^1 \frac{\tau\varphi(\tau)\,d\tau}{\sqrt{\tau^2 - z^2}}.$$

Substituting the expression for $w(a, z)$ into (2.4) we obtain

$$F(s) = \frac{\pi}{2}\int_0^1 \tau\varphi(\tau) J_0(s\tau)\,d\tau.$$

The second equation of system (2.3) is satisfied identically for any function $\varphi(t)$. Substituting $F(s)$ as the defining function in the first equation of system (2.3) and carrying out the required transformations, as was done in [4], we obtain the integral Fredholm equation of the second kind

$$\varphi_1(t) - \int_0^1 \varphi_1(\tau) K(\tau, t)\,d\tau = F(t), \quad 0 \leqslant t \leqslant 1, \tag{2.5}$$

where

$$\varphi_1(t) = \varphi(t)\sqrt{t}\,\tau_0^{-1};\; K(\tau, t) = 2\sqrt{\tau t}\int_0^\infty s\,[1/2 - saI_2(sa)K_2(sa)]\,J_0(st)J_0(s\tau)\,ds;$$

$$F(t) = \frac{4}{\pi}\sqrt{t}\left\{\int_0^t \frac{\tau_1(z)\,dz}{\sqrt{t^2 - z^2}} - \int_0^\infty s^2 aI_2(sa)K_1(sa)T_0(s)J_0(st)\,ds\right\}.$$

3. The Main Features of the Stresses and Discussion of the Results. The solutions obtained have the property that the stresses have a singularity at the vertex of the crack of the order of $r_0^{-1/2}$, where $r_0 \ll 1$ is the distance external to the vertex of the crack (see Fig. 1). The conditions for limiting equilibrium of the crack are completely defined by the coefficients of the stress intensity K_{I}, K_{II}, and K_{III} for these singularities. We will obtain the stress intensity coefficient of the problems considered assuming that the solutions of the Fredholm equations (1.5) and (2.5) are known. For twisting of a cylindrical crack the stress $\sigma_{r\varphi}$ can be written in the form

$$\sigma_{r\varphi}^{(1)}(r, z) \simeq \int_0^\infty s^2 aK_2(sa)I_2(sr)\cos(sz)\,ds\int_0^1 \tau\varphi(\tau)J_0(s\tau)\,d\tau + \ldots\,. \tag{3.1}$$

Assuming that $a - r = \varepsilon \ll 1$, for $s \gg 1$ we have

$$s^2 aK_2(sa)I_2(sr) \simeq e^{-\varepsilon s}[(2as)^{-1} + O(s^{-2})]. \tag{3.2}$$

In the inner integral we will also distinguish the principal part

$$\int_0^1 \tau\varphi(\tau)J_0(s\tau)\,d\tau \simeq \frac{1}{s}\varphi(1)J_1(s). \tag{3.3}$$

Taking into account only the principal terms from (3.2) and (3.3) in (3.1) (it can be shown that the remaining terms play no part in the formation of the singularity), we obtain [5]

$$\sigma_{r\varphi}(r, z) \simeq \frac{\varphi(1)}{2}\int_0^\infty e^{-\varepsilon s}J_1(s)\cos(sz)\,ds = \frac{\varphi(1)}{2}\left[1 - (2r_0)^{-1/2}\cos\frac{\varphi}{2}\right]$$

or, reverting back to the dimensional quantities, we have

$$\sigma_{r\varphi} \simeq \frac{\varphi_1(1)}{2}\tau_0\left[1 - \sqrt{\frac{l}{2}}\cos\frac{\varphi}{2}r_0^{-1/2}\right] = \tau_0\frac{\varphi_1(1)}{2} - \frac{K_{\mathrm{III}}}{\sqrt{r_0}}\cos\frac{\varphi}{2}. \tag{3.4}$$

Proceeding in a similar way as in the axisymmetrical case, and omitting the fairly lengthy calculations, we obtain asymptotic equations for the components of the stress tensor, which hold in a small neighborhood of the vertex of the crack and agree with the equations given in [6]

$$\sigma_{rr}(r, z) = \frac{p_0\sqrt{l}}{2\sqrt{2r_0}}\left\{\delta\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\cos\frac{3\varphi}{2} - \varphi_1(1)\cos\frac{\varphi}{2}\left[1 + \sin\frac{\varphi}{2}\sin\frac{3\varphi}{2}\right]\right\},$$

$$\sigma_{zz}(r, z) = \frac{p_0\sqrt{l}}{2\sqrt{2r_0}}\left\{-\delta\sin\frac{\varphi}{2}\left[2 + \cos\frac{\varphi}{2}\cos\frac{3\varphi}{2}\right] - \varphi_1(1)\cos\frac{\varphi}{2}\left[1 - \sin\frac{\varphi}{2}\sin\frac{3\varphi}{2}\right]\right\}$$

$$\sigma_{rz}(r, z) = \frac{p_0\sqrt{l}}{2\sqrt{2r_0}}\left\{\delta\cos\frac{\varphi}{2}\left[1 - \sin\frac{\varphi}{2}\sin\frac{3\varphi}{2}\right] - \varphi_1(1)\sin\frac{\varphi}{2}\cos\frac{\varphi}{2}\cos\frac{3\varphi}{2}\right\}.$$

Hence

$$K_{\mathrm{I}} = -\frac{\varphi_1(1)p_0\sqrt{l}}{2\sqrt{2}}, \quad K_{\mathrm{II}} = \frac{p_0\sqrt{l}}{2\sqrt{2}}\delta, \tag{3.5}$$

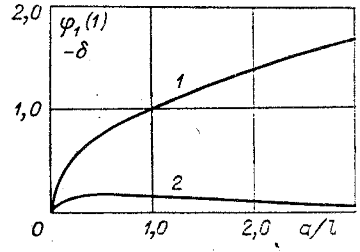

Fig. 2

where δ and $\varphi_1(1)$ are given by Eqs. (1.5).

In the limiting case as $a \to \infty$ and $\tau_1(z) = \tau_2(z) \equiv 1$, we obtain from Eq. (2.5) $\varphi_1(1) = 2$, and from (3.4) we obtain $K_{III} = \tau_0\sqrt{l}/\sqrt{2}$, which corresponds to the solution of the problem of the equilibrium of an isolated crack of length $2l$ under antiplane deformation conditions.

In the axisymmetrical case from Eqs. (1.5) as $a \to \infty$ we obtain $\varphi_1(1) = 2$, $\psi_1(t) = 0$, from (3.5) we obtain $K_I = -p_0\sqrt{l/2}$; $K_{II} = 0$, which corresponds to the solution for an isolated crack under plane deformation conditions.

The integral Fredholm equations (1.5) and (2.5) were evaluated numerically on a computer. As an example, we show the results obtained in evaluating the system of integral equations (1.5) for $\nu = 0.3$ graphically in Fig. 2. Here curves 1 and 2 correspond to the variation of $\varphi_1(1)$ and $(-\delta)$ as a function of the ratio a/l. For small values of a/l the quantity $\varphi_1(1)$ can be regarded as stable and its values can be approximated quite well by the expression

$$\varphi_1(1) = 1.095\sqrt{a/l}.$$

When $a/l \ll 1$ the solution of the problem reduces to the solution of the problem of the equilibrium of a semi-infinite cylindrical crack, for which, using Rice's method [6], we obtain

$$K_I^2 + K_{II}^2 = (2\pi)^{-1}\frac{p_0^2 a}{1-\nu^2} \tag{3.6}$$

or

$$\varphi_1^2(1) + \delta^2 = \frac{4}{\pi(1-\nu^2)}\frac{a}{l}.$$

Using this relation we can refine the dependence of $-\delta$ (curve 2) on a/l for small values of a/l, where it is difficult to make numerical calculations on a computer.

Figure 3 shows the results of a numerical evaluation of Eq. (2.5) assuming that

$$-\tau_2(z) = \tau_1(z) = \begin{cases} 1, & 1 - l_0' \leqslant z \leqslant 1, \\ 0, & 0 \leqslant z < 1 - l_0'. \end{cases}$$

Lines 1–5 represent the variation in $a/l \cdot \varphi_1(1)$ as a function of a/l for $l_0/l = 1.0, 0.9, 0.8, 0.5$, and 0.2, respectively. If $a/l \gg 1$, we have $a/l \cdot \varphi_1(1) \sim \sqrt{a/l}$. In the other limiting case for $a/l \ll 1$, using Rice's method [6] we obtain

$$K_{III} = \frac{2\tau_0 l_0}{\sqrt{2\pi}\, a^{1/2}} = \frac{M}{\pi\sqrt{2\pi}\, a^{5/2}}, \tag{3.7}$$

where $M = 2\pi a^2 \tau_0 l_0$ is the moment produced by the stress τ_0 acting on a section of length l_0. Curves 1–5, with an accuracy of from 1% to 5% (depending on the value of the ratio l_0/l) for $a/l \le 2$ can be approximated by an expression similar to that obtained in [7]

$$\frac{a}{l}\varphi_1(1) = \frac{4}{\sqrt{\pi}}\sqrt{\frac{a}{l}}\frac{l_0}{l}\left[1 - 0.047\frac{a}{l}\right]. \tag{3.8}$$

In Fig. 3 the lines 6–10 represent the results of numerical evaluation of Eq. (2.5) assuming that $\tau_2(z) \equiv 0$, and $\tau_1(z)$ is the same as above for $l_0/l = 1.0, 0.9, 0.8, 0.5$, and 0.2, respectively. In this case for $a/l \gg 1$ the quantity $a/l \cdot \varphi_1(1)$ asymptotically approaches the value $1 - 2/\pi \cdot \arcsin(1 - l_0)$, and for $a/l \le 1$ with an accuracy of to within 2% can be approximated by the equation

$$\frac{a}{l}\varphi_1(1) = \frac{4}{\sqrt{\pi}}\sqrt{\frac{a}{l}}\left[\frac{l_0}{l} + 0.096\frac{a}{l}\right].$$

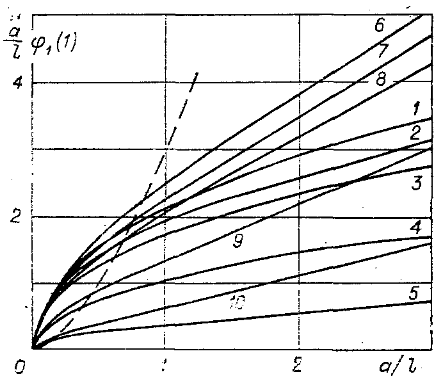

Fig. 3

In the problem of twisting for z = 0 the stress $\sigma_{z\varphi} = 0$, i.e., the plane z = 0 is free. The problem can be interpreted as follows: A cylindrical crack (core) is drilled perpendicular to a free surface, while the stress $\sigma_{r\varphi}$ represents the friction between the instrument and the rock. Taking as the criterion of fracture [3]

$$(1-\nu)\left[K_{\mathrm{I}}^{2}+K_{\mathrm{II}}^{2}\right]+K_{\mathrm{III}}^{2}=2\mu\frac{\gamma}{\pi}, \tag{3.9}$$

where γ is the surface energy arriving per unit of free surface of the body, and assuming that the quantities γ, μ, τ_0, a are specified, using curve 1 in Fig. 3 we can determine the length of the first piece broken off. Considering only the twisting ($K_I = K_{II} = 0$), we obtain from (3.4) and (3.9)

$$\frac{a}{l}\varphi_1(1)=\frac{4}{\tau_0}\sqrt{\frac{\mu\gamma}{\pi a}}\left(\frac{a}{l}\right)^{3/2}=k_0\left(\frac{a}{l}\right)^{3/2}. \tag{3.10}$$

This relationship is represented by the dashed curve in Fig. 3 (in this case the constant $k_0 = 3$). The point of intersection of this curve and curve 1 defines the length of the first piece $a/l_1 = 0.73$ or $l_1 = 1.37a$.

Consider the following idealization of the process. We will assume that the instrument sinks a distance l_0, and we will then assume that the tangential stresses act only on this part l_0, while on the length of the part which has broken off l_1 they are zero (it rotates together with the cutting instrument), and the overall length in this case will be $l = l_0 + l_1$. In order to obtain the size of the second piece, in this case it is sufficient to equate (3.8) (where we have taken only the principal term) and (3.10)

$$k_0(a/l)^{3/2}=(4/\sqrt{\pi})(a/l)^{1/2}\cdot l_0/l,$$

whence $l_2 = l_0 = k_0\sqrt{\pi}/4\cdot a = 1.33a$. Obviously this process can be extended further.

In the general case when $K_I \neq 0$ and $K_{II} \neq 0$, i.e., there is side compression, in the limiting case when $a/l \ll 1$ we can estimate the effect of the value of the side pressure p_0 on the size of the pieces broken off. To do this we substitute the asymptotic expressions (3.6) and (3.7) into (3.9) and obtain

$$\frac{l_0^2}{a^2}+\frac{1}{4(1+\nu)}\frac{p_0^2}{\tau_0^2}=\frac{\gamma\mu}{a\tau_0^2}.$$

Hence we see that the maximum size of the pieces broken off when drilling the core is obtained when there is no side compression ($p_0 = 0$). As p_0 increases the size of the piece decreases and approaches zero when $p_0^2 = 4(1+\nu)\gamma\mu a^{-1}$.

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CONSTRUCTION OF THE CREEP EQUATIONS FOR MATERIALS WITH DIFFERENT EXTENSION AND COMPRESSION PROPERTIES

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Constructional materials of light alloys such as aluminum-magnesium and titanium possess different tensile properties for tension and compression. Whereas the "instantaneous" elastoplastic properties may differ only slightly, the difference in the properties under prolonged action (e.g., the duration up to fracture) may reach several orders of magnitude [1]. Figure 1 shows a diagram of the creep of VT-9 titanium alloy at a temperature of 400°C with different combinations of tension (compression) and twisting at a constant stress

$$\sigma_i = (\sigma^2 + 3\tau^2)^{1/2} = 72.5 \ \mathrm{kg/mm^2}$$

in the form of the time-dependence $A = \sigma\varepsilon + \tau\gamma$. The marks on the diagrams correspond to the marks of the scheme of the stressed state of the plane $\sigma - \sqrt{3}\tau$. It can be seen from the diagram that the intensity of the creep process with σ_i = const decreases as the stress state changes from pure tension to pure shear and compression. Here for comparison we show two diagrams, namely, pure tension with $\sigma_i = 71$ kg/mm² (points 1) and twisting of a thin-walled tubular specimen with $\sigma_i = 77.5$ kg/mm² (points 2), the intensity of the creep process of which is the same.* The example given clearly illustrates the need to construct a theory which would enable one to describe creep in complex media.

One of the first attempts to describe creep in media with different resistance to tension and compression is described in [2], in which the actual stresses are replaced by "reduced" stresses, and a theory is constructed assuming similarity between the deviators of the rates of deformation and the "reduced" stresses. This method has not been developed any further, and in practice even simple problems lead to quite complicated equations [3].

Another approach is to construct creep equations in the form of a dependence of the "equivalent rate of deformation" η_e on the "equivalent stress" σ_e, where the intensity of the rate of deformation $\eta_i = ({}^2/{}_3\eta_{kl}\eta_{kl})^{1/2}$ is usually taken as η_e, while σ_e is considered as a function of the stress tensor invariants. The creep equation is supplemented by the law of flow (e.g., by the gradient $\eta_{kl} = k\partial\sigma_e'/\partial\sigma_{kl}$, where σ_e' is not always the same as σ_e) [4, 5].

Attempts have been to construct equations assuming the existence of a potential creep function which depends on the stress tensor invariants and scalar parameters of the strengths [6-13]. The potential function is assumed to be both smooth [6-8, 12] and piecewise-smooth [9, 11, 13], and equations have also been constructed with more general assumptions [14].

When constructing defining equations the different resistance to tension and compression is taken into account by introducing into σ_e, in addition to the second invariant of the stress deviator, one of the odd invariants: In a number of papers preference is given to the first invariant of the stress tensor [6, 9, 11, 15, 16-18], while in others preference is given to the third invariant of the stress deviator [7, 8, 12, 14]. Although it is not our purpose to make a more detailed review of the papers in this field, we will illustrate the most typical approaches to constructing defining equations containing in addition either the first or third invariants of the stress tensor by using the example of the creep of OT-4 titanium alloy at a temperature of 475°C and different combinations of tension-twisting and compression-twisting.

* N. G. Torshenov participated in these experiments.

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